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History of Low-Order Equivalent Systems for Aircraft Flying Qualities

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Introduction

FROM the beginnings of the evolution of flight, there evolved an appreciation of the advantages of an aircraft that was easy for the pilot to fly. Among the insights for which they are famous is the Wright brothers' realization that when the ability to balance and steer became understood, then aviation would truly have arrived. As the balance between stability and control became better understood, aircraft emerged with sufficiently good flying (or handling) qualities to allow air-to-air combat, reliable passenger transport, and a host of other uses.

As aeronautical engineers examined the measurable factors that seemed to bestow good flying qualities, they followed the urge of all engineers, which is to write down those lessons, rules of thumb, and guidelines that make the job easier the next time. From the observations of these engineers, and especially from dedicated research using special-purpose variable-stability aircraft, the community began to document characteristics that were preferred for good flying qualities, along with characteristics that would indicate poor qualities.

Most of the early flying-qualities work involved essentially steady-state characteristics like stick force variation with steady, trimmed speed. Dynamic behavior was not so well understood, though early aviators did comprehend the slow tradeoff between altitude and speed in Lanchester's phugoid mode. They also knew all too well the sinister origins of the slow divergence called the spiral mode. The quick lateral-directional oscillations of the Dutch roll were commonly experienced. Appreciation of the rigid-body modes of motion greatly improved in the first half of the 20th century, including the faster lateral roll mode and the quick angle-of-attack oscillations of the short-period mode. By the middle of the 20th century, engineers realized that the time constants, damping ratios, and natural frequencies of all of these modes could be correlated with flying qualities, good or bad. The stage was set for formal requirements, in the form of military specifications, to define the good and bad values of these modal parameters.

The 1960s: Inception

By the late 1960s, the dynamic requirements of flying-qualities specifications had evolved into a remarkably elegant form. The language of the specification drew on the language of linear classical control theory, which was becoming well understood in the middle of the 20th century. The lessons from hundreds of research experiments, many using variable stability aircraft, were combined to allow the rigid-body modes of motion—the longitudinal short-period and phugoid, and the lateral-directional dutch roll, spiral and roll modes—to be specified quite closely.

However, the introduction of feedback control to augment the aircraft's stability and control characteristics, which by this time was becoming state of the art, posed some problems. Implementation of the feedback introduced some higher-order effects, and the responses of these emerging designs could not be fully characterized using the simple modes of motion. In addition to the sensor and actuator dynamics mandated by the feedback control, filters in the command path, forward and feedback loop added to the complexity of the system.

Seeing the need to gather data on augmented aircraft with high-order responses, the U.S. Air Force Flight Dynamics Laboratory sponsored two key research activities. One was a ground-based simulation by Parrag[1] and the other was a later in-flight simulation by DiFranco.[2]

In DiFranco's experiment, "the airplane short period response to step stick force inputs can be reasonably well represented by a time delay and an equivalent second-order response" (see Fig. 1), and "Pilot ratings and PIO [Pilot-Induced Oscillation] ratings … correlate reasonably well with a computed delay parameter."

Chalk et al.[3] later described DiFranco's results on the delay parameter as "rather startling and in direct contradiction to [earlier NACA experiments in which an overly sensitive aircraft was improved by filtering the pilot's input to the control surfaces, using a filter with lag equal to or lower than the short-period frequency]… even moderate lags can cause very pronounced pilot-induced oscillations…the allowable lag decreases with increasing [short-period frequency]." The PIOs seen in DiFranco's experiment had not been seen at all in Parrag's ground-based experiment, leading DiFranco to state, "In evaluating PIO tendencies, ground simulator results are not conservative and can be very misleading." Chalk et al. incorporated DiFranco's results into the specification by restricting the allowable phase shift at the short-period frequency. In effect, this requirement allows larger delays for slower configurations. (The comparison between flight and ground simulation still warrants study. For example, Brandon et al.[4] found that "pilot input magnitudes, especially in pitch, tend to be much greater in the simulator than in flight," but obtained PIOs in both.)

DiFranco's results were viewed as somewhat specialized because his control dynamics were Butterworth filters, which mimic delays in their response characteristics. Therefore his observation regarding the addition of a time delay to a second-order response was thought to have little generality for the emerging flight control systems with a greater variety of higher-order dynamics. Also, his results were surprising and contradicted the common wisdom.

The 1970s: Validation

Other influencial studies were those of Stapleford et al.[5] and Craig et al.[6] These studies described, using transport and fighter examples, how selecting a single mode from the higher-order response is misleading, and instead demonstrated how to "match the amplitude and phase characteristics over the frequency range of interest with an equivalent second-order system." The equivalent terms matched the general behavior of the response far better than dominant modes. The focus of these two studies however was on the mid-frequency response, and so no high-frequency terms or delays were included.

So in setting the stage, we have at this point an increasing awareness that 1) high-order terms can strongly affect handling qualities; 2) low-order equivalent systems are a candidate approach; 3) the

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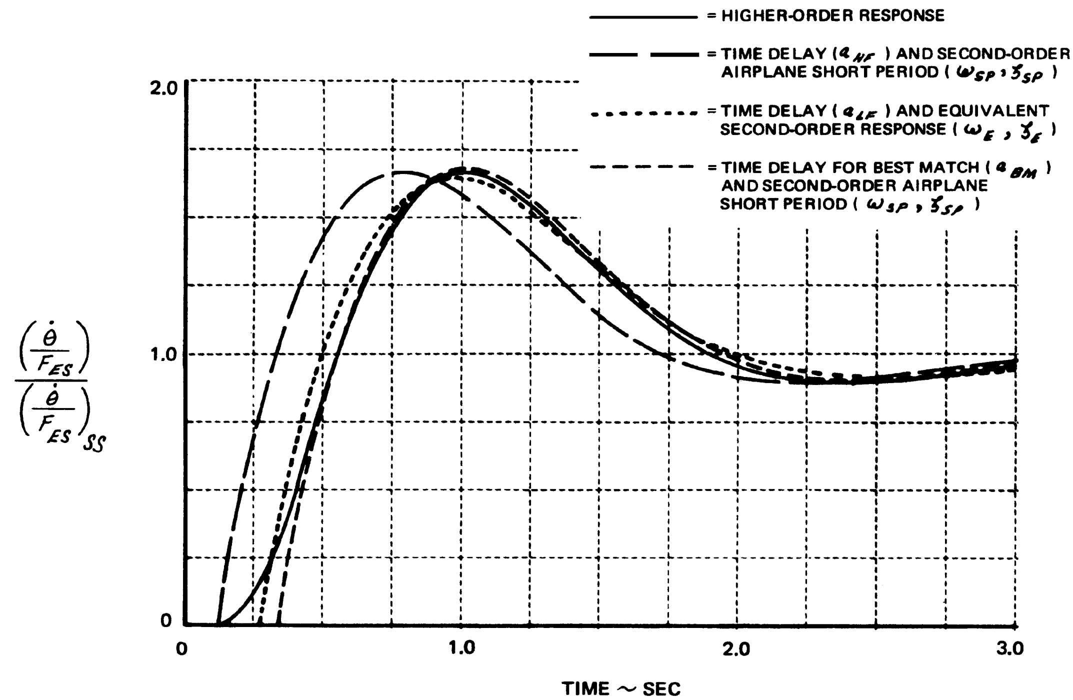

Fig. 1 Example of DiFranco's high-order response, second-order equivalent, and time delay.

phase lags, or delay-like behavior of high-frequency terms could cause PIOs; 4) ground-based simulation might overlook the piloting effects of delays; and 5) with increasing short-period frequency, the allowable lag decreased.

There were in that period a number of aircraft manufacturers competing for business with the U.S. government. Ten manufacturers, for example, sent representatives to one flying-qualities workshop.[7] Charles B. Westbrook at the U.S. Air Force Flight Dynamics Laboratory, whose Control Criteria Branch had contracted with Cornell Aeronautical Laboratory, Inc., to draft the new flying-qualities specification,[3] let several contracts to manufacturers compare the new requirements with the known flying qualities of one of their own aircraft. In the case of McDonnell Aircraft Co. (McAir) in St. Louis, the aircraft was the F-4 model.[8,9] Westbrook's stated strategy was not only to calibrate the technical accuracy of the new specification, but also to establish within each manufacturing organization a cadre of individuals and their management with a detailed knowledge of, and appreciation for, the specification. One of the concerns of the F-4 study was the difficulty in applying modal criteria when the aircraft had additional modes. At McAir, the late Chester Miller, Chief of Aerodynamics, began funding flying-qualities research under the company's formal Independent Research and Development program. Similar investments began at other manufacturers.

The research at McAir began with a plan to investigate current flying-qualities methods for high-order dynamics. Four methods were planned for evaluation, beginning with a quick evaluation of the equivalent system approach using a frequency-matching computer program that was written for the purpose. Fortunately, in addition to DiFranco and Parrag's data just mentioned, the results from a new experiment had just become available.

The Neal–Smith experiment[10] incorporated the lessons learned from Cornell's earlier DiFranco and Parrag experiments and drew ideas from the pilot modeling work of Systems Technology. Neal and Smith examined contemporary flight control systems and incorporated their generic characteristics in a data base of 51 configurations, which were flown in demanding piloting tasks. The control system dynamics were representative of current practice and were not restricted to Butterworth forms. Needless to say, the Neal–Smith data were ideal for McAir's planned analysis.

Neal and Smith attempted low-order equivalent system (LOES) analysis of their data using time response analog matching, which proved unreliable. At McAir, we thought that using frequency response matching, and doing it with a digital computer program incorporating a search or optimization method, might be more precise and repeatable. We began with a rank one search method from Kujawski[11] that worked for a few test cases. The search routine minimized the differences in gain and phase at 20 equispaced frequencies, on a logarithmic scale:

$$M = \Sigma(G_{\mathrm{HOS}} - G_{\mathrm{LOES}})^2 + K\,\Sigma(\phi_{\mathrm{HOS}} - \phi_{\mathrm{LOES}})^2 \qquad (1)$$

or

$$M = \sum_{i-1}^{20}\left(G_i^2 + 0.02\phi_i^2\right) \qquad (2)$$

The transfer function used for the low-order pitch response was

$$\frac{q}{F_x} = \frac{K_q(s + L_\alpha)e^{-tt}}{s^2 + 2\varsigma\omega s + \omega^2} \qquad (3)$$

This is the classical short-period form, with an equivalent delay that we found matched quite well the accrued high-frequency phase lags, even though they were not of the Butterworth form.

With the gain in decibels and the phase in degrees, a weighting function K of around 0.02 is used. Equation (2) was evaluated at 20 discrete equispaced intervals on a log scale, between the minimum and maximum specified frequencies for the matching, generally 0.1 and 10.0 radians per second. This range appeared adequate for longitudinal pitch response to pilot force commands.

At this point, the method was a somewhat hand-crafted affair with few features to assist with repeated running, for example. LaManna was undaunted and embarked on analysis of all of the Neal–Smith configurations. He quickly determined that the rank-one method was not well suited to our problem, however, and substituted a Rosenbrock modified pattern search that allowed use of a penalty function. The penalty function restrained the roots of the equivalent system to the left-half plane because we were dealing exclusively with stable dynamics. We tried several search methods for this problem, and the Rosenbrock method appeared the most robust and adequate for use by, for example, Brulle and Moran's F-15 study.[12]

As in the DiFranco analysis, the Neal–Smith configurations with larger delays, or phase lags, had PIOs. What surprised us was that for the more demanding piloting tasks in the Neal–Smith data, very small delays degraded the ratings noticeably. Level 1 flying qualities were precluded if the delay exceeded about 0.1 s (actually 0.07 s). The allowable delay did not seem to be a function of frequency, as in the preceding item 5).

There were more surprises. Overall, the high quality of the matches was unexpected. The configurations were not designed to be second-order like at all, and yet even the worst matches captured the nature of the responses. The best matches were essentially indistinguishable.[13]

Some configurations in the Neal–Smith database included mid-frequency lags that partially canceled the lead caused by the L_α term in the numerator of the transfer function of pitch rate to elevator command. The computer program attempted to match these responses by moving the numerator term to higher frequencies. We did not quite know what to do with cases like these, because if we fixed the numerator L_α in the pitch response the match was generally quite poor, leaving in question the poor values of frequency and time delay that were calculated. If on the other hand we included the L_α numerator term in the matching process, the match was excellent, but the L_α was unrealistically large. For these configurations, whose response is not aircraft-like and whose flying qualities were poor, we reinterpreted the control-anticipation-parameter (CAP) requirements by translating the large L_α into a large effective n/α and pointing out that the corresponding frequency now became too low. In this way, we showed that the response was too sluggish. Later, this "galloping L_α" treatment was judged too much of a stretch from the physical dynamics, and finally an approach that combined pitch matching with normal load factor matching was adopted for official use in the military specifications. This method for conventional configurations was the same as fixing the L_α numerator in a pitch-rate-only match.

When we looked at the entire Neal–Smith database of equivalent systems, we concluded that the method correlated well with the modal requirements and produced an estimate of equivalent delay, a new parameter that would help with the design specification, and analysis of the high-order systems that were planned for the next generation of aircraft. Augmented, high-order responses could be evaluated and understood using our experience with the requirements of the specification. Again, we were surprised to find that the Neal–Smith data consisted of essentially classically behaved aircraft, but some configurations had an equivalent delay that degraded the ratings. We also observed that no configuration with a poor match had good handling qualities.

Members of a separate group at McAir were also surprised and were especially skeptical regarding the general applicability of these initial equivalent system results. As a test case, they derived equivalent systems for a highly augmented F-4 research aircraft. Its longitudinal control system was designed using a C* criterion[14] consisting of a time-history envelope, without regard to classical modal parameters like short period damping and frequency. Even so, when they used time-history responses to pilot square wave inputs to contrast the high- and low-order responses they found that the equivalent systems were excellent for the best cases and quite acceptable even for the worst-matched cases.[15]

Many subsequent workers exhibited healthy similar skepticism regarding the LOES approach. In the specific case of LOES, there were several concerns that 1) the method would only apply at best to a small subset of high order responses; 2) artifacts of the matching method (weighting, frequency distribution and range, uniqueness of the matches, etc.) would negate the repeatability and value of the method; and 3) the equivalent delay term did not affect the frequency region of piloted crossover, casting doubt on the correlations with pilot rating. Certainly the author and coinvestigators shared all of these concerns, which were tackled in many of the later references in this paper.

The LOES approach became widely accepted only when a number of government, research, and manufacturing organizations tried the method and obtained reasonable results for a very wide range of aircraft configurations. A key consideration for manufacturers was that the Neal–Smith database reflected a limited range of basic modal short-period parameters at two flight conditions. Criteria based solely on this database therefore tended to enforce this narrow range of modal characteristics. The LOES approach instead highlights the Neal–Smith data as a subset of a much broader database in which a wide range of dampings and frequencies and acceleration sensitivities would still result in good handling qualities. When they came to appreciate this less restrictive view, manufacturers (in the United States at least) happily adopted the technique.

By the mid-1970s, digital flight control systems were emerging from the promising research results of aircraft like the Digitac A-7, for example, and the NASA DFBW F-8, which first flew in 1972. These digital systems added sample-and-hold, processing latency, and antialiasing filters to the complexity. Against this background, development of the F-18 digital fly-by-wire system was well under way. The LOES approach, which by this time was being applied to lateral-directional dynamics,[16] was also applied to the longitudinal responses of the F-18 aircraft (Ringland[17] and Hodgkinson[18]), which was encountering flying-qualities problems caused mostly by excessive delays. The following year the Landing Approach High Order System (LAHOS) experiment[19] made use of the LOES experience by evaluating short-period-plus-delay configurations and provided invaluable background data for the F-18 development.

LOES methods were being proposed[7,20] for use in the military specification. Johnston and Hodgkinson[21] validated the use of LOES for fighter landing tasks using the LAHOS data and these results and others[22,23] added to the evidence that the method could be suitable for formal specification of flying qualities.

At this point, there was an increasing acceptance that low-order models might be an acceptable way of dealing with other high-order problems, including those of vertical/short-takeoff-and-landing (V/STOL) aircraft, beginning with an analysis of the YAV-8B.[24] And the more theoretical aspects of determining a match attracted interest, including time-domain approaches to the match.[25] A comparison of the Neal–Smith handling-qualities criterion with equivalent system predictions[26] showed a remarkable correlation between short-period frequency and the pilot lead parameter. This study was an early example of direct comparisons of different criteria.

The 1980s: Maturation

Application to V/STOL configurations continued.[27] Shafer[28] demonstrated identification of equivalent systems from flight data using a maximum likelihood time-response method. Both time-response and frequency-response methods have been used over the years; however, the combination of a fast Fourier transform and an equivalent system match probably accounts for more of the literature.

Other studies evaluated the LOES approach,[29] while Radford et al. directly compared the method with others.[30] V/STOl applications continued.[31]

The question of precision of match had been tackled by specifying the mismatch function in Fig. 2. Values of 10 or so were considered excellent, and 100 was considered an indication of a poor match. Determination of the correctness of the equivalent however mostly depended on the eye of the beholder, which is a time-honored engineering approach but rather imprecise where military specifications are concerned. Bode envelopes of mismatch were developed[32] from available high-order data by identifying how much dynamics could be added to a configuration without causing a difference noticeable to the pilot. The resulting envelopes are very interesting, partly because they exhibit a neck, presumably at the piloted crossover frequency for the data used, and partly because they are unexpectedly large. In general, the engineer's eye would reject equivalents, which barely fell within the envelopes. For this reason, the author thought the envelopes would be of marginal use; however, they have a way of reemerging regularly for a variety of uses; including validation for simulation and for model-following precision, etc.

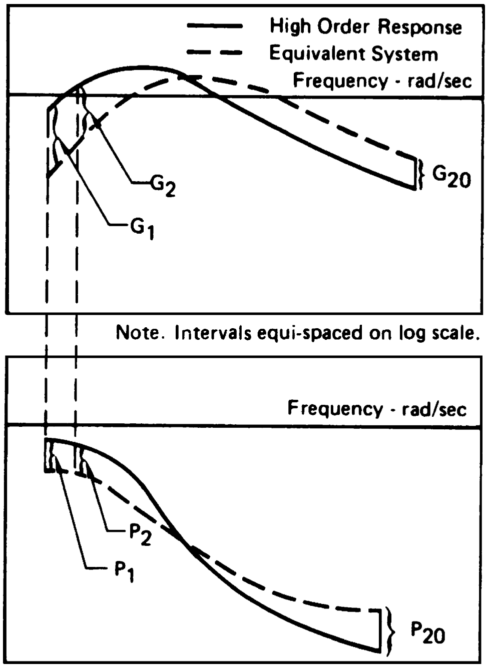

Fig. 2 Definition of mismatch function for LOES.

This period marked more official interest from U.S. government agencies.[33–36] LOES was defined as the basic approach to all of the modal requirements of the specification, and a new requirement on equivalent time delay was added. V/STOL work continued.[37] It had been agreed by this time that the numerator term in Eq. (3) should be fixed at the aircraft natural value, either directly or by including the normal load factor response in the match along with pitch rate.

Of alternatives to LOES emerging at that time, the bandwidth criterion appeared particularly promising and a good counterpoint to the equivalent systems approach. Whereas the LOES method produces several parameters that together give a picture of the handling qualities, requiring use of a computer program, the bandwidth method required choosing a single bandwidth directly from the Bode plot. It was based on the simplest interpretation of pilot-in-the-loop manual control. Unfortunately, early correlations of the Neal–Smith data with bandwidth alone were quite poor. Addition of equivalent delay as a second dimension to bandwidth greatly improved correlation. As already mentioned, time delay did not quite fit into the conventions of manual control theory, and the supposition has since been that delay interferes with the ability of the pilot to increase the pilot-in-the-loop bandwidth when required because the delay approximates the rapid degradation in phase lag as urgency (i.e., frequency of excitation) increases. However, adding equivalent delay to the bandwidth criterion negated much of the simplicity of the bandwidth method, and so Wood proposed a simple measure directly from the Bode plot, which approximated equivalent delay as closely as possible.[38] An experiment to investigate directly the equivalence between low- and high-order systems[39] produced a wealth of data, including comparisons between actual and equivalent delays. Part of the experiment involved attempts to cancel, at a specific frequency, the phase lag caused by delay. Unfortunately this approach proved to be ineffective, or even dangerous.

The main outcome of this experiment, however, was to validate the equivalence in handling qualities of the LOES dynamics. With that validation, some interesting possibilities emerged. An important one was the possibility of flight simulation using a low-order model, and Carpenter[40] simulated translation rate systems using low-order models that were easy to simulate and debug, as well as being directly related to the parameters being correlated with pilot opinion. This study was one of the first that extended the LOES concept beyond conventional fixed-wing dynamics.

The equivalence between actual delays and equivalent delays also suggested simulations simply using pure delays rather than combinations of high-order terms, and Berry et al.[41] performed a landmark study to demonstrate handling qualities degradation caused by delay. An AGARD meeting in 1982 was not only an opportunity for review of progress in the United States[42,43] but also an opportunity to see some European views on the subject. Neuhuber et al.[44] used the LOES approach both for criteria and as the controlled element in a pilot-in-the-loop analysis.

By this time, experience with LOES had reached the point where Mitchell and Hoh[45] documented some concerns about the method. The chief concern was with configurations with pitch lead compensation. The apparent gain resonance can unrealistically produce a low equivalent damping for these configurations because the lead term in the match is by convention fixed at the aircraft L_α value. Of course, the mismatch is higher for these configurations, and an inspection of the match indicates what the problem is. The bandwidth criterion can often recognize this characteristic with a shelf-like gain response and a correspondingly low gain-limited bandwidth. However, visual inspection is a good idea with this criterion too because there is always the possibility of a pathological response with the undesirable shelf but (just barely) an excellent bandwidth. In today's world of large-scale batch processing of many flight conditions, for example, visual inspection does not always occur as it should.

In spite of these concerns, Hoh et al. produced a detailed proposal for the Military Standard and Handbook[46] that included LOES for the modal criteria. About that time, Hoh et al. also produced a study of vehicles with sufficient control effectors and control power to allow independent control of all six degrees of freedom.[47] This document contains a table that effectively defines the LOES for many of the possible modes of motion. Thus the LOES approach was being applied not only to V/STOL problems,[48] but also to the simulation of other unconventional aircraft dynamics.[49]

Kisslinger and Krachmalnick at this time provide a fascinating glimpse[50] into the integration of flying qualities into a modern fly-by-wire design, not least of which was initial confusion over the term "delay."

The synergy with the bandwidth criterion led to proposed modifications to the bandwidth,[51] and Melnyk et al.[52] pointed out that a Bode envelope proposed by Nelson of Northrop was restrictive compared with equivalent systems and the control anticipation parameter (Fig. 3).

Bischoff and Palmer[53] documented lateral-directional LOES models, and Wood[54] found the slope of rating degradation with time

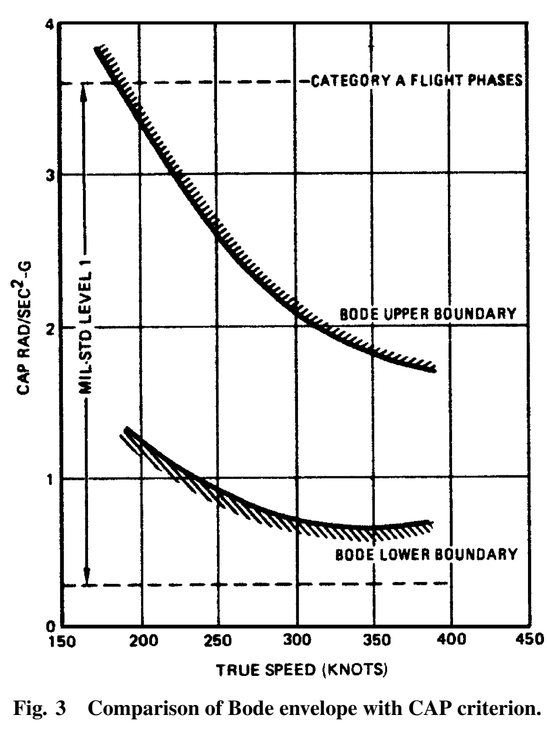

Fig. 3 Comparison of Bode envelope with CAP criterion.

delay to be one rating point per 40 ms in the simulator vs 20 ms in flight. The idea of incorporating the LOES rules into an expert-aided system was explored by Joshi et al.[55]

One question with all the criteria mentioned, including LOES, is whether to include the feel system in the equivalent system. The specifications, based on research data for which the feel system was very fast, favored inclusion of the feel system. Smith and Sarrafian[56] pointed out that slow feel systems do not degrade handling as much as the equivalent delay would suggest. Bailey[57] returned to the question of feel system inclusion, and Potsdam and Hodgkinson[58] suggested partial inclusion of feel system lag in the equivalent delay.

Bacon and Schmidt[59,60] suggested a mathematical approach to determining LOES. Hodgkinson et al.[61] and Hodgkinson and Hodgkinson[62] published what was to be the first of a number of assertions that agility potential would not be attained if the handling qualities were not adequate,[63–65] whereas Colgren put LOES in the larger context of mathematical order reduction.[66] In these analyses, equivalent system concepts were helpful for insight into the behavior of high-order systems, and Chody et al.[63] extended Hodgkinson and Hodgkinson's assertion to equivalent potential agility.

The 1990s: Proliferation

Hodgkinson et al.[67] presents some lessons learned in evaluating high-order dynamics in the previous years.

Buchacher et al.[68] presented interesting procedural rules and problems with determining equivalent systems. They "outsmarted" the mismatch envelopes (without difficulty in view of their large size) and pointed out the necessity to measure normal load factor at the center of rotation, a requirement sometimes not mentioned in the literature. They also outsmarted the default frequency distribution by contriving a low-damped example. And they pointed out a significant difference in predicted handling levels using phase delay vs equivalent delay. Mitchell[69] reminded readers that the phase delay was born as a simply measured approximation to equivalent delay and that differences can indeed exist. An AGARD working group on highly augmented aircraft[70] pointed out the general similarity between all of the proposed criteria restricting high-frequency phase lag.

For the author, the transition from the 1980s to the 1990s was a transition from fighter applications, with the challenges of augmented fly-by-wire systems and the Advanced Tactical Fighter competition, to transport applications. But the transport world was just embarking on fly-by-wire control systems. Unfortunately there was no Neal–Smith or LAHOS database for transport dynamics to aid development of the MD-11 and C-17. Therefore, work began on developing a flying-qualities database, using variable stability aircraft when possible. This was part of a technology transfer from the fighter world,[71] where the LOES approach had become relatively standard for fighter dynamics, even as a basis for nonlinear lateral-directional problems.[72] At this time, frequency-domain parameter identification particularly suited to equivalent systems was emerging, chiefly from helicopter applications.[73] These methods were helpful in validating simulation models.[74,75] Also from a helicopter background, Padfield et al. described the influence of flying qualities on agility.[64,65]

Marchand et al.'s very interesting AGARD paper from 1993 (Ref. 76) described some difficulties with the LOES method as applied to the European Fighter Aircraft (EFA). Generally, highly augmented aircraft, particularly those with extra control surfaces like canards, have very low-order-appearing, decoupled responses,[47] but apparently EFA had high-order-appearing dynamics in both longitudinal and lateral responses. The longitudinal pitch LOES, for example, could fit either the low-frequency response or the pitch resonance, but not both. In the face of these responses, their decision to abandon the LOES method is understandable. However, in the databases described in this present paper, inability to achieve a match is associated with poor handling qualities. Another AGARD activity, this time a working group, included another look at agility.[65]

The gathering of data for transports continued,[77–79] whereas for fighters Moorhouse and Citurs[80] describe an inverse-equivalent-systems technique in which the control system gains are chosen to force the high-order dynamics to approximate a specified low-order response. By specifying the low-order model every tenth of a Mach number, they were able to achieve a design with smoothly scheduled gains across the flight envelope of a modified F-15. They also, based on fixed-base simulation of pitch tracking, used a lag-lead prefilter to alter the apparent L_α of the pitch response. Though this approach, as they recognized, degrades the normal load factor response, the degradation is relatively small. (A brief analysis by the author confirms that the equivalent system match is worsened, but not dramatically.) Unfortunately, later unpublished experience has shown that the apparently minor degradation in the flight-path response caused, in the case of one pilot, significant PIOs in formation flying. A summary of work on transport flying qualities[81] did not mention this approach for the larger aircraft.

Along the lines of the inverse method of Moorhouse and Citurs, Tischler et al.[82,83] proposed a sophisticated control design method called CONDUIT, incorporating essentially any criteria desired by the designer, including equivalent systems along with their envelopes. Use of Tischler's CIFER identification program was also described in a paper by Hodgkinson et al.[84]

Persistent PIOs in the U.S. fleet led to intensified research on PIO in this time period. Most of the research centered on criteria, which advertised themselves as PIO criteria; however, the phase-delay parameter in the bandwidth criterion was clearly very successful in predicting PIOs as a result of linear high-order dynamics, and so by association equivalent delay would also be successful. This was confirmed by Hodgkinson et al.[85] About this time, the first general text on handling qualities appeared,[86] containing a section on equivalent systems.

Rate limiting of control surfaces was emerging as a cause of PIO, and Mitchell et al.[87] demonstrated how to extend the bandwidth criterion to these nonlinearities. Mitchell also in unpublished data examined equivalent systems for rate-limited configurations, using a fast Fourier identification from flight frequency sweeps. Very briefly put, increasing rate limiting increases the mismatch along with the equivalent delay as one would expect. Both indications would steer the designer away from significant limiting.

Also in quite recent work, Hodgkinson and Mitchell[88] summarized much of the work on equivalent systems and bandwidth, including the prediction of PIOs. For those of us who suspected the heyday of LOES was over, A NATO/RTO summary of flight control system (FCS) best practices[89] stated: ". . . use of these specifications is currently out of favor . . . most FCS problems come from characteristics that violate the specification requirements as written." There still remained much valuable work to be done on parameter identification, as exemplified by Morelli's approach.[90,91] Hodgkinson et al.[92] confirmed that LOES, though not advertised explicitly as a PIO criterion, can certainly guide designers away from PIO.

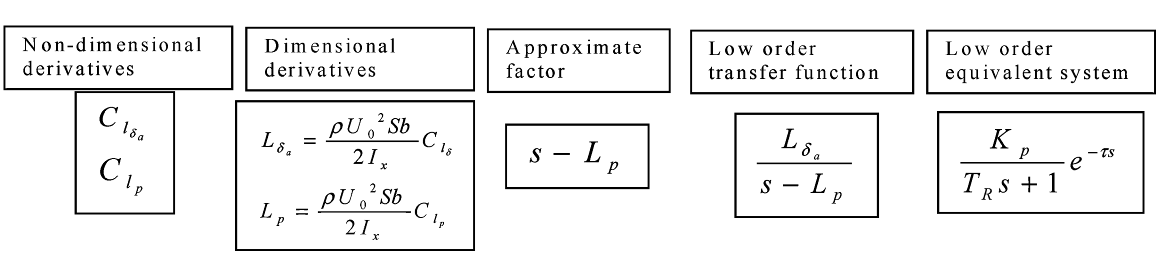

Fig. 4 Continuum of parameters for lateral response.

The LOES approach does not treat the aircraft dynamics as a generic controlled element or plant, but employs parameters and responses familiar to flight dynamicists. As shown in the simple lateral example of Fig. 4, LOES is merely an evolution of our efforts to understand and codify today's aircraft responses. It builds on the work of the Wrights, Gates, Bryant, McRuer, and others who laid the groundwork of flight dynamics.

Conclusions

The preceding has been a relatively brief and necessarily personal review of the development of the low-order equivalent system method. Many relevant publications, unfortunately, have been omitted for reasons of space. In conclusion, we can look back at the history and offer some overall comments.

Has the method fallen out of favor, as suggested by the Flight Control Best Practices report in 2000? Was it a stop-gap until something else came along? Evidently several other criteria did come along, but as our single example of the Bode envelope shows, the other criteria tended to be more stringent without clear justification for the stringency. As for falling out of favor, the Best Practices report gives a clear clue that use of the equivalent system method, including nonlinear characteristics as required by the specifications, would have avoided problems in several control development efforts.

The insight required for and endowed by the equivalent system process remains a worthwhile component of flying qualities design and analysis.

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